Monotonicity under Knowledge

Epistemic Possibility in Logic and Conversation

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Outline

Interaction between knowledge, belief and epistemic possibility

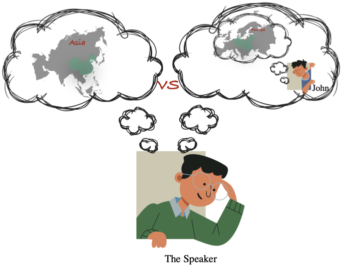
- Puzzle 1: Tension between free choice and monotonicity
- Puzzle 2: Know-might and believe-might

Main ingredients of the analysis

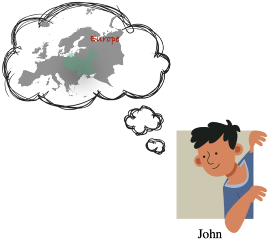
- Free choice as pragmatic enrichment in BSML
- Expressive analysis of epistemic possibility
- Plausibility model for belief in epistemic logic
- Factivity as presupposition

A framework for epistemic possibility under knowledge and belief

Knowledge, belief and epistemic possibility



John *knows* China *might* be in Europe



John *believes* China *might* be in Europe

Free choice disjunction

- Free choice principle

$$\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$$

  - Deontic free choice:

(1) a. Zhangsan may go to the restaurant or to the cinema.
    b. $\rightsquigarrow$ Zhangsan may go to the restaurant and he may go to the cinema

(Kamp1973)

  - Epistemic free choice:

(2) a. Zhangsan might be in Beijing or in London
    b. $\rightsquigarrow$ Zhangsan might be in Beijing and he might be in London

(Zimmermann2000)

Free choice under knowledge and belief

- Free choice under knowledge: $[K]\blacklozenge(\alpha \vee \beta) \rightsquigarrow [K]\blacklozenge\alpha \wedge [K]\blacklozenge\beta$

(3) a. Lisi knows Zhangsan might be in Beijing or in London
    b. $\rightsquigarrow$ Lisi knows Zhangsan might be in Beijing, and Lisi knows Zhangsan might be in London

- Free choice under belief: $[B]\blacklozenge(\alpha \vee \beta) \rightsquigarrow [B]\blacklozenge\alpha \wedge [B]\blacklozenge\beta$

(4) a. Lisi believes Zhangsan might be in Beijing or in London
    b. $\rightsquigarrow$ Lisi believes Zhangsan might be in Beijing, and Lisi believes Zhangsan might be in London

Puzzle 1: free choice vs monotonicity

a. China is in Asia (and not in Europe) p

b. John knows that China might be in Asia. $[K]\blacklozenge p$

upward monotonicity

c. John knows that China might be in Asia or Europe. $[K]\blacklozenge(p \vee \neg p)$

free choice principle

d. John knows that China might be in Europe. $[K]\blacklozenge\neg p$

factivity of knowledge

e. China might be in Europe. $\blacklozenge\neg p$

- The puzzle:

  # China is not in Europe, but China might be in Europe.

Epistemic contradictions

Why contradictory?

Philosophers have observed the infelicity of the following sentence (Wittgenstein1953, Veltman1996, Yalcin2007, Mandelkern2019):

(5) # It is not raining but it might be raining

- Epistemic contradiction: $\neg\phi \wedge \Diamond\phi \vDash \bot$
- Non-factivity of *might*: $\Diamond\phi \nvDash \phi$

(Yalcin2007)

Puzzle 2: know-might vs believe-might

- Believing epistemic contradictions

(6) # John believes that it is raining but he believes that it might not be raining.

It is always incoherent to believe an epistemic contradiction (see Beddor&Goldstein2017)

- Replace the *believe* in the latter clause with *know*:

(7) John believes that it is raining but he knows that it might not be raining.

However, if 'know' entails 'believe', this is puzzling[1].

[1] This puzzle is from Matthew Mandelkern

Strategy for puzzle 1: free choice in BSML

- In Bilateral state-based modal logic (BSML, Aloni2022), free choice effects can be derived **only** for pragmatically enriched formulas: $[\Diamond(\phi \vee \psi)]^+ \vDash \Diamond\phi \wedge \Diamond\psi$

$$[K]\blacklozenge p$$

$$\Downarrow \qquad\qquad \not\Downarrow$$

$$[K]\blacklozenge(p \vee \neg p) \qquad\qquad [K][\blacklozenge(p \vee \neg p)]^+$$

$$\not\Downarrow \qquad\qquad \Downarrow$$

$$[K]\blacklozenge\neg p$$

Strategy for puzzle 2: plausibility model in epistemic logic

In logic, a view of belief follows the intuition that we believe those things that hold in the ‘best’ worlds epistemically accessible to us, see (vanBenthem2006, BaltagSmets2006) etc.

- A plausibility ordering ranks the epistemic states, and hence gives much finer gradations.
- Doxastic states are those most plausible states.

Plausibility model

- A plausibility model is a tuple $M = \langle W, \sim, \leq_w, V \rangle$
  - $\sim$ is an equivalence relation, and gives rise to equivalence classes $[w]$ for each world $w \in W$.
  - $\leq_w$ is plausibility relations for all worlds $w \in W$. Intuitively, $v \leq_w v'$ says that at world w, the agent thinks that v is at least as plausible as v'.
- The semantics for the doxastic formula is defined as:

$$M, w \vDash [B]\phi \text{ iff } M, v \vDash \phi \text{ for all } v \in Min_{\leq_w}([w])$$

where $Min_{\leq_w}([w]) := \{v \in [w] \mid \forall v' \in [w] : v' \leq_w v \Rightarrow v \leq_w v'\}$

- Notice that: on this account knowledge entails belief.

Epistemic extension for BSML

Language

The language $\mathcal{L}_{\mathcal{E}}$ is the propositional language enriched with the non-emptiness atom NE, epistemic operators and presupposition.

$$\varphi ::= p \mid \neg\varphi \mid \varphi_{\varphi} \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid [B]\varphi \mid [I]\varphi \mid \blacklozenge\varphi \mid \mathrm{NE}$$

- $[I]\varphi$ means that according to the information the agent has, φ is true.
- φ_{φ} indicates that φ presupposes φ.
- $\blacklozenge$ is epistemic possibility.
- Knowledge as a derived notion:

$$[K]\phi := ([I]\phi)_{\phi}$$

Semantics

- An epistemic model for $\mathcal{L}_{\mathcal{E}}$ is a tuple $\mathcal{M} = \langle W, \sim, \leq_w, V \rangle$.
  - $\sim$ is an equivalence relation on W,and gives rise to the equivalence class $[w]$:=$\{v \mid v \sim w\}$.
  - $\leq_w$ is the plausibility ordering among possible worlds. $Min_{\leq_w}([w]) := \{v \in [w] \mid \forall v' \in [w] : v' \leq_w v \Rightarrow v \leq_w v'\}$
- An information state s is a subset of W: $s \subseteq W$. Formulas in our language are interpreted with respect to an information state: $\mathcal{M}, s \vDash \phi$.
- The interpretation $M, s \vDash \phi$ stands for "formula ϕ is assertable in s" and $M, s \dashv \phi$ stands for "ϕ is rejectable in s"

Semantics

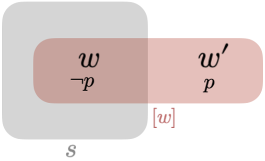
$$
\begin{array}{lll}
\mathcal{M}, s \vDash p & \text{iff} & \forall w \in s : V(w,p) = 1 \\
\mathcal{M}, s \Dashv p & \text{iff} & \forall w \in s : V(w,p) = 0 \\
\\
\mathcal{M}, s \vDash \neg\phi & \text{iff} & \mathcal{M}, s \Dashv \phi \\
\mathcal{M}, s \Dashv \neg\phi & \text{iff} & \mathcal{M}, s \vDash \phi \\
\\
\mathcal{M}, s \vDash \phi \vee \psi & \text{iff} & \exists t, t' {:} t \cup t' {=} s \text{ and } \mathcal{M}, t \vDash \phi \text{ and } \mathcal{M}, t' \vDash \psi \\
\mathcal{M}, s \Dashv \phi \vee \psi & \text{iff} & \mathcal{M}, s \Dashv \phi \text{ and } \mathcal{M}, s \Dashv \psi \\
\\
\mathcal{M}, s \vDash \phi \wedge \psi & \text{iff} & \mathcal{M}, s \vDash \phi \text{ and } \mathcal{M}, s \vDash \psi \\
\mathcal{M}, s \Dashv \phi \wedge \psi & \text{iff} & \exists t, t' {:} t \cup t' {=} s \text{ and } \mathcal{M}, t \Dashv \phi \text{ and } \mathcal{M}, t' \Dashv \psi \\
\\
\mathcal{M}, s \vDash \texttt{NE} & \text{iff} & s \neq \varnothing \\
\mathcal{M}, s \Dashv \texttt{NE} & \text{iff} & s = \varnothing
\end{array}
$$

Semantics

$$
\begin{array}{lll}
\mathcal{M}, s \vDash \blacklozenge\phi & \text{iff} & \exists s' \subseteq s \text{ and } s' \neq \varnothing \text{ s.t. } \mathcal{M}, s' \vDash \phi \\
\mathcal{M}, s \Dashv \blacklozenge\phi & \text{iff} & \mathcal{M}, s \Dashv \phi \\
\\
\mathcal{M}, s \vDash [I]\phi & \text{iff} & \forall w \in s : [w] \vDash \phi \\
\mathcal{M}, s \Dashv [I]\phi & \text{iff} & \forall w \in s\text{: there is a non-empty subset } t \subseteq [w] \text{ and } t \Dashv \phi \\
\\
\mathcal{M}, s \vDash [B]\phi & \text{iff} & \forall w \in s,\ Min_{\leq_w}([w]) \vDash \phi \\
\mathcal{M}, s \Dashv [B]\phi & \text{iff} & \forall w \in s,\ Min_{\leq_w}([w]) \Dashv \phi
\end{array}
$$

Reflexivity runs into problems for factivity

- Reflexivity is not enough to derive factivity. We have $[I]p \vDash p$
- But, we have problems with $[I]\blacklozenge p \vDash \blacklozenge p$:



$$s \vDash [I]\blacklozenge p, \text{ but } s \nvDash \blacklozenge p$$

- $\blacklozenge p$ within the scope of $[I]$ is interpreted with respect to $[w]$. Whereas $\blacklozenge p$ is interpreted with respect to s.

Factivity as presupposition

- To derive the factivity of knowledge with *might*-sentences, we define knowledge in term of presupposition and information:

$$[K]\phi := ([I]\phi)_\phi$$

$$\begin{array}{lll} \mathcal{M}, s \vDash \phi_\psi & \text{iff} & s \vDash \phi \text{ and } s \vDash \psi \\ \mathcal{M}, s \dashv \phi_\psi & \text{iff} & s \dashv \phi \text{ and } s \vDash \psi \end{array}$$

- Intuitively, 'a knows ϕ' not only means a has information of ϕ, it also presupposes that ϕ is true. Then we have

$$[K]\blacklozenge p \vDash \blacklozenge p$$

- But also we have the projection of factive inferences under negation:

$$\neg[K]\blacklozenge p \vDash \blacklozenge p$$

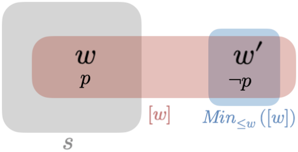
(8) a. John doesn't know Susan might be in Bergen
b. ⇒ Susan might be in Bergen

From knowledge to belief

- In our framework, the entailment from knowledge to belief holds only for the ♦-free fragment of the language:

$$[K]\phi \vDash [B]\phi, \text{ if } \phi \text{ is a } \blacklozenge\text{-free formula}$$

- But $[K]\blacklozenge p \nvDash [B]\blacklozenge p$.



$$s \vDash [K]\blacklozenge p, \text{ but } s \nvDash [B]\blacklozenge p$$

- This gives us a solution to Puzzle 2.

Summery of results

- Epistemic contradiction

  $p \wedge \blacklozenge \neg p \vDash \bot$

- Non-factivity of epistemic possibility

  $\blacklozenge p \nvDash p$

- Free choice under attitude verbs (Puzzle 1)

  $[K]/[B]/[I][\blacklozenge(\phi \vee \psi)]^{+} \vDash [K]/[B]/[I]\blacklozenge\phi \wedge [K]/[B]/[I]\blacklozenge\psi$

- Believing epistemic contradictions (Puzzle 2)

  $[B]p \wedge [B]\blacklozenge\neg p \vDash \bot$

  $[B]p \wedge [K]\blacklozenge\neg p \nvDash \bot$

- Duality

  $[I]\blacklozenge\phi \Leftrightarrow \langle I \rangle \phi$

  $[B]\blacklozenge\phi \Leftrightarrow \langle B \rangle \phi$

- Factivity of knowledge with might

  $[K]\blacklozenge\phi \vDash \blacklozenge\phi$

- Factive inference projects under negation

  $\neg[K]\blacklozenge\phi \vDash \blacklozenge\phi$

Conclusion

- Two puzzles
  - Puzzle 1: tension between free choice and monotonicity
  - Puzzle 2: know-might and believe might
- Main ingredients in our analysis:
  - Free choice as pragmatic enrichment in BSML (Aloni2022)
  - Expressive analysis of epistemic possibility (Veltman1996, Yalcin2007)
  - Plausibility model for belief in epistemic logic (VanBenthem2006, BaltagSmets2006)
  - Factivity as presupposition (Lasersohn2009, SpectorEgre2015 etc.)
- Future work
  - Dynamics: dynamic semantics perspective vs dynamic epistemic logic; first person perspective vs third person perspective
  - Multi-agency: impact on factivity; speaker-oriented interpretation vs subject-oriented interpretation

Thank You!!!