Monotonicity under Knowledge

Epistemic Possibility in Logic and Conversation

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Outline

Interaction between knowledge, belief and epistemic possibility as expressed by English *might*

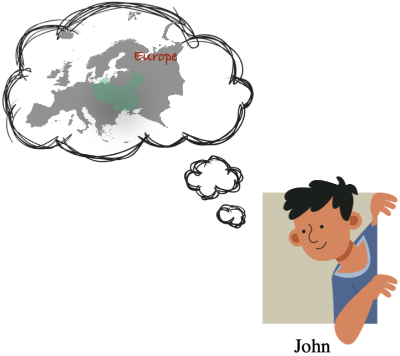
- Puzzle 1: Tension between free choice and monotonicity
- Puzzle 2: Know-might and believe-might

Main ingredients of the analysis

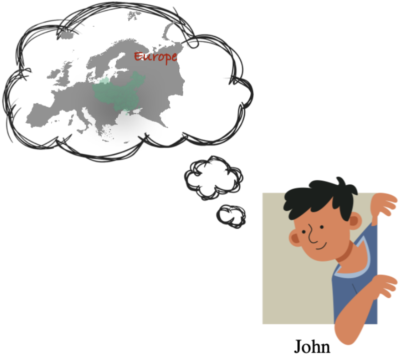
- Free choice as pragmatic enrichment in BSML
- Non truth-conditional analysis of *might*
- Plausibility model for belief in epistemic logic
- Factivity as presupposition

A framework for epistemic possibility under knowledge and belief

Knowledge, belief and epistemic possibility

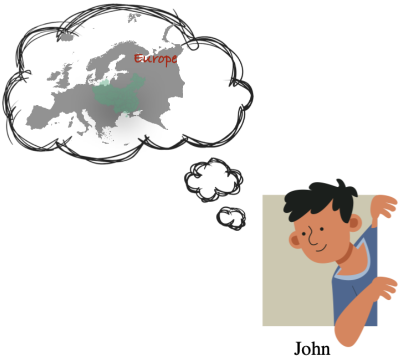


Knowledge, belief and epistemic possibility



John believes that China might be in Europe

Knowledge, belief and epistemic possibility



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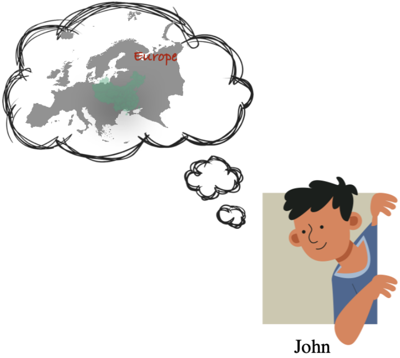
Knowledge, belief and epistemic possibility



John believes that China might be in Europe

John knows that China might be in Europe

Factivity of Knowledge: $[K]\phi \vDash \phi$

Free choice inference

- Free choice principle

$$\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$$

  - Deontic free choice:

(1) a. Zhangsan may go to the restaurant or to the cinema
b. $\rightsquigarrow$ Zhangsan may go to the restaurant and he may go to the cinema

(Kamp1973)

  - Epistemic free choice:

(2) a. Zhangsan might be in Beijing or in London
b. $\rightsquigarrow$ Zhangsan might be in Beijing and he might be in London

(Zimmermann2000)

Free choice under knowledge and belief

- Free choice under knowledge: $[K]\blacklozenge(\alpha \vee \beta) \rightsquigarrow [K]\blacklozenge\alpha \wedge [K]\blacklozenge\beta$[1]

(3) a. Lisi knows Zhangsan might be in Beijing or in London
    b. $\rightsquigarrow$ Lisi knows Zhangsan might be in Beijing, and Lisi knows Zhangsan might be in London

- Free choice under belief: $[B]\blacklozenge(\alpha \vee \beta) \rightsquigarrow [B]\blacklozenge\alpha \wedge [B]\blacklozenge\beta$

(4) a. Lisi believes Zhangsan might be in Beijing or in London
    b. $\rightsquigarrow$ Lisi believes Zhangsan might be in Beijing, and Lisi believes Zhangsan might be in London

[1] The operator $\blacklozenge$ represents epistemic possibility MIGHT in this talk

Puzzle 1: free choice vs monotonicity

a. China is in Asia (and not in Europe)	p	*premise*
b. John knows that China might be in Asia.	$[K]\blacklozenge p$	*premise*
c. John knows that China might be in Asia or Europe.	$[K]\blacklozenge(p \vee \neg p)$	*upward monotonicity, b*
d. John knows that China might be in Europe.	$[K]\blacklozenge\neg p$	*free choice principle, c*
e. China might be in Europe.	$\blacklozenge\neg p$	*factivity of knowledge, d*

Puzzle 1: free choice vs monotonicity

a. China is in Asia (and not in Europe)	p	*premise*
b. John knows that China might be in Asia.	$[K]\blacklozenge p$	*premise*
c. John knows that China might be in Asia or Europe.	$[K]\blacklozenge(p \vee \neg p)$	*upward monotonicity, b*
d. John knows that China might be in Europe.	$[K]\blacklozenge\neg p$	*free choice principle, c*
e. China might be in Europe.	$\blacklozenge\neg p$	*factivity of knowledge, d*

- The puzzle:

(5) # China is not in Europe, but China might be in Europe.

Epistemic contradictions

Why contradictory?

Infelicity of the following sentence (Wittgenstein1953, Veltman1996, Yalcin2007, Mandelkern2019):

(6) # It is not raining but it might be raining

A tension between two desiderata:

- Epistemic contradiction: $\neg\phi \land \blacklozenge\phi \vDash \bot$
- Non-factivity of *might*: $\blacklozenge\phi \nvDash \phi$

(Yalcin2007)

Puzzle 2: know-might vs believe-might

- Believing epistemic contradictions

(7) # John believes that it is raining but he believes that it might not be raining.

It is always incoherent to believe an epistemic contradiction (see Beddor&Goldstein2017)

- If we replace *believe* in the latter clause then the result is coherent:

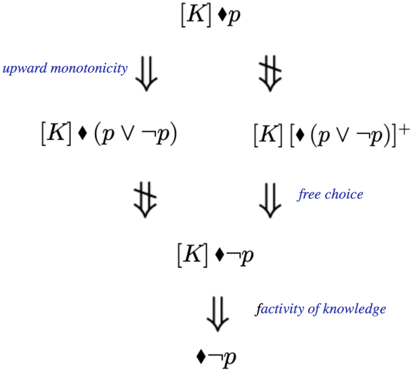
(8) John believes that it is raining but he knows that it might not be raining.

However, if 'know' entails 'believe', this is puzzling[2].

[2] This puzzle is from Matthew Mandelkern (pc)

Strategy for puzzle 1: free choice in BSML

- In Bilateral state-based modal logic (BSML, Aloni2022, Anttila et al. 2022) free choice effects can be derived **only** for pragmatically enriched formulas: $[\Diamond(\phi \vee \psi)]^{+} \vDash \Diamond\phi \wedge \Diamond\psi$



Strategy for puzzle 2: plausibility model in epistemic logic

In logic, a view of belief follows the intuition that we believe those things that hold in the 'best' worlds epistemically accessible to us, see (vanBenthem2006, BaltagSmets2006) etc.

- A plausibility ordering ranks the epistemic states, and hence gives much finer gradations.
- Knowledge interpreted w.r.t epistemic states, and belief interpreted w.r.t the best worlds epistemically accessible.
- Doxastic states are those most plausible worlds.

Plausibility model

- A plausibility model is a tuple $M = \langle W, \sim, \leq_w, V \rangle$
  - $\sim$ is an equivalence relation, and gives rise to equivalence classes $[w]$ for each world $w \in W$.
  - $\leq_w$ is plausibility relations for all worlds $w \in W$. Intuitively, $v \leq_w v'$ says that at world w, the agent thinks that v is at least as plausible as v'.
- The semantics for the doxastic formula is defined as:

$$M, w \vDash [B]\phi \text{ iff } M, v \vDash \phi \text{ for all } v \in Min_{\leq_w}([w])$$

where $Min_{\leq_w}([w]) := \{v \in [w] \mid \forall v' \in [w] : v' \leq_w v \Rightarrow v \leq_w v'\}$

- Notice that: on this account knowledge entails belief.

Epistemic extension of BSML

Language

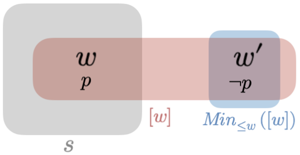
$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \texttt{NE} \mid [B]\varphi \mid [I]\varphi \mid \blacklozenge\varphi \mid \varphi_{\varphi}$$

- $\texttt{NE}$ is the non-emptiness atom from team logic (Yang&V$\ddot{a}\ddot{a}$n$\ddot{a}$nen2017)
- $[B]\varphi$ means that the agent believes φ.
- $[I]\varphi$ means that according to the information the agent has, φ is true.
- $\blacklozenge$ represents epistemic possibility MIGHT.
- φ_{φ} indicates that φ presupposes φ.
- Knowledge as a derived notion:

$$[K]\phi := ([I]\phi)_{\phi}$$

Semantics

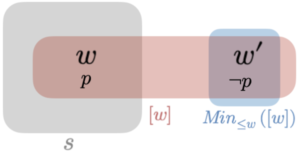
- An epistemic model for $\mathcal{L}_{\mathcal{E}}$ is a tuple $\mathcal{M} = \langle W, \sim, \leq_w, V \rangle$.
  - $\sim$ is an equivalence relation on W,and gives rise to the equivalence class $[w]{:=}\{v \mid v \sim w\}$.
  - $\leq_w$ is the plausibility ordering among possible worlds. $Min_{\leq_w}([w]) := \{v \in [w] \mid \forall v' \in [w] : v' \leq_w v \Rightarrow v \leq_w v'\}$
- An information state s is a subset of W: $s \subseteq W$. Formulas in our language are interpreted with respect to an information state: $\mathcal{M}, s \vDash \phi$.
- The interpretation $M, s \vDash \phi$ stands for "formula ϕ is assertable in s" and $M, s \dashv \phi$ stands for "ϕ is rejectable in s"



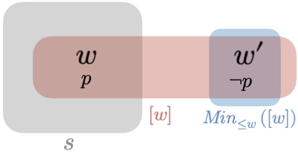
Semantics

$$
\begin{array}{lll}
\mathcal{M}, s \vDash p & \text{iff} & \forall w \in s : V(w,p) = 1 \\
\mathcal{M}, s \dashv p & \text{iff} & \forall w \in s : V(w,p) = 0 \\
\\
\mathcal{M}, s \vDash \neg\phi & \text{iff} & \mathcal{M}, s \dashv \phi \\
\mathcal{M}, s \dashv \neg\phi & \text{iff} & \mathcal{M}, s \vDash \phi \\
\\
\mathcal{M}, s \vDash \phi \vee \psi & \text{iff} & \exists t, t' : t \cup t' = s \text{ and } \mathcal{M}, t \vDash \phi \text{ and } \mathcal{M}, t' \vDash \psi \\
\mathcal{M}, s \dashv \phi \vee \psi & \text{iff} & \mathcal{M}, s \dashv \phi \text{ and } \mathcal{M}, s \dashv \psi \\
\\
\mathcal{M}, s \vDash \phi \wedge \psi & \text{iff} & \mathcal{M}, s \vDash \phi \text{ and } \mathcal{M}, s \vDash \psi \\
\mathcal{M}, s \dashv \phi \wedge \psi & \text{iff} & \exists t, t' : t \cup t' = s \text{ and } \mathcal{M}, t \dashv \phi \text{ and } \mathcal{M}, t' \dashv \psi \\
\\
\mathcal{M}, s \vDash \texttt{NE} & \text{iff} & s \neq \varnothing \\
\mathcal{M}, s \dashv \texttt{NE} & \text{iff} & s = \varnothing
\end{array}
$$

Semantics

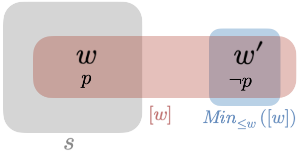


Semantics



$$\mathcal{M}, s \vDash \blacklozenge\phi \quad \text{iff} \quad \exists s' \subseteq s \text{ and } s' \neq \varnothing \text{ s.t. } \mathcal{M}, s' \vDash \phi$$
$$\mathcal{M}, s \dashv \blacklozenge\phi \quad \text{iff} \quad \mathcal{M}, s \dashv \phi$$

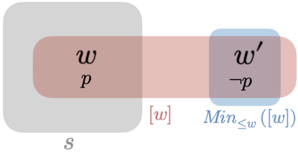
Semantics



$$
\begin{array}{lll}
\mathcal{M}, s \vDash \blacklozenge\phi & \text{iff} & \exists s' \subseteq s \text{ and } s' \neq \varnothing \text{ s.t. } \mathcal{M}, s' \vDash \phi \\
\mathcal{M}, s \dashv \blacklozenge\phi & \text{iff} & \mathcal{M}, s \dashv \phi
\end{array}
$$

$$
\begin{array}{lll}
\mathcal{M}, s \vDash [I]\phi & \text{iff} & \forall w \in s : [w] \vDash \phi \\
\mathcal{M}, s \dashv [I]\phi & \text{iff} & \forall w \in s\text{: there is a non-empty subset } t \subseteq [w] \text{ and } t \dashv \phi
\end{array}
$$

Semantics



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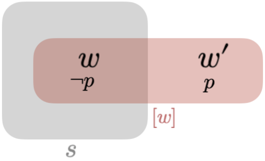
\begin{array}{lll}
\mathcal{M}, s \vDash \blacklozenge\phi & \text{iff} & \exists s' \subseteq s \text{ and } s' \neq \varnothing \text{ s.t. } \mathcal{M}, s' \vDash \phi \\
\mathcal{M}, s \dashv \blacklozenge\phi & \text{iff} & \mathcal{M}, s \dashv \phi \\
\\
\mathcal{M}, s \vDash [I]\phi & \text{iff} & \forall w \in s : [w] \vDash \phi \\
\mathcal{M}, s \dashv [I]\phi & \text{iff} & \forall w \in s\text{: there is a non-empty subset } t \subseteq [w] \text{ and } t \dashv \phi \\
\\
\mathcal{M}, s \vDash [B]\phi & \text{iff} & \forall w \in s,\ Min_{\leq_w}([w]) \vDash \phi \\
\mathcal{M}, s \dashv [B]\phi & \text{iff} & \forall w \in s,\ Min_{\leq_w}([w]) \dashv \phi
\end{array}
$$

Reflexivity runs into problems for factivity

- If $\sim$ is reflexive then
- We have $[I]p \vDash p$. But, we have problems with $[I]\blacklozenge p \vDash \blacklozenge p$:



$$s \vDash [I]\blacklozenge p, \text{ but } s \nvDash \blacklozenge p$$

- $\blacklozenge p$ within the scope of $[I]$ is interpreted with respect to $[w]$. Whereas $\blacklozenge p$ is interpreted with respect to s.

Factivity as presupposition

- To derive the factivity of knowledge with *might*-sentences, we define knowledge in term of presupposition and information:

$$[K]\phi := ([I]\phi)_\phi$$

$$\begin{array}{lll} \mathcal{M}, s \vDash \phi_\psi & \text{iff} & s \vDash \phi \text{ and } s \vDash \psi \\ \mathcal{M}, s \dashv \phi_\psi & \text{iff} & s \dashv \phi \text{ and } s \vDash \psi \end{array}$$

- Intuitively, 'a knows ϕ' not only means a has information of ϕ, it also presupposes that ϕ is true. Then we have

$$[K]\blacklozenge p \vDash \blacklozenge p$$

- But also we have the projection of factive inferences under negation:

$$\neg[K]\blacklozenge p \vDash \blacklozenge p$$

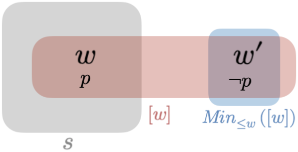
(9) a. John doesn't know Susan might be in Bergen
b. ⇒ Susan might be in Bergen

From knowledge to belief

- In our framework, the entailment from knowledge to belief holds only for the ⧫-free fragment of the language:

$$[K]\phi \vDash [B]\phi, \text{ if } \phi \text{ is a } ⧫\text{-free formula}$$

- But $[K]⧫p \nvDash [B]⧫p$.



$$s \vDash [K]⧫p, \text{ but } s \nvDash [B]⧫p$$

- This gives us a solution to Puzzle 2.

Summary of results I

- Epistemic contradiction

  $p \wedge \blacklozenge\neg p \vDash \bot$

- Non-factivity of epistemic possibility

  $\blacklozenge p \nvDash p$

- Free choice inference

  $[\blacklozenge(\phi \vee \psi)]^{+} \vDash \blacklozenge\phi \wedge \blacklozenge\psi$

- Free choice under attitude verbs (Puzzle 1)

  $[K]/[B]/[I][\blacklozenge(\phi \vee \psi)]^{+} \vDash [K]/[B]/[I]\blacklozenge\phi \wedge [K]/[B]/[I]\blacklozenge\psi$

- Believing epistemic contradictions (Puzzle 2)

  $[B]p \wedge [B]\blacklozenge\neg p \vDash \bot$

  $[B]p \wedge [K]\blacklozenge\neg p \nvDash \bot$

- Duality

  $[I]\blacklozenge\phi \Leftrightarrow \langle I\rangle\phi$, $\langle I\rangle\phi := \neg[I]\neg\phi$

  $[B]\blacklozenge\phi \Leftrightarrow \langle B\rangle\phi$, $\langle B\rangle\phi := \neg[B]\neg\phi$

Summary of results II

- Factivity of knowledge with might

  $[K]\blacklozenge\phi \vDash \blacklozenge\phi$

- Factive inference projects under negation

  $\neg[K]\blacklozenge\phi \vDash \blacklozenge\phi$

- Neg-raising for belief but not for knowledge

  $\neg[B]\phi \vDash [B]\neg\phi$

  $\neg[K]\phi \nvDash [K]\neg\phi$

(10) a. John doesn't believe that it is raining $\Rightarrow$ John believes that it is not raining

b. John doesn't know that it is raining $\nRightarrow$ John knows that it is not raining

Conclusion

- Two puzzles
  - Puzzle 1: tension between free choice and monotonicity
  - Puzzle 2: know-might and believe might
- Main ingredients in our analysis:
  - Free choice as pragmatic enrichment in BSML (Aloni2022)
  - Expressive analysis of epistemic possibility (Veltman1996, Yalcin2007)
  - Plausibility model for belief in epistemic logic (VanBenthem2006, BaltagSmets2006)
  - Factivity as presupposition (Lasersohn2009, SpectorEgre2015 etc.)
- Future work
  - Dynamics: dynamic semantics perspective vs dynamic epistemic logic; first person perspective vs third person perspective
  - Multi-agency: impact on factivity; speaker-oriented interpretation vs subject-oriented interpretation

Thank You!!!