Knowing and Believing an Epistemic Possibility

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- # John knows that China might be in Europe

Knowledge, belief and epistemic possibilities

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    b. ⇝ Daiyu might be in Suzhou and Daiyu might be in Jinling.

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- Well-known properties of knowledge and belief:
  - Upward monotonicity:

    (3) a. Every dog barks. □ Fido barks.
    b. Daiyu knows/believes that every dog barks. □ Daiyu knows/believes that Fido barks.

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  - Factivity of knowledge: KNOW p □ p

Outline

Interaction between knowledge, belief and epistemic possibilities as expressed by English ***might***

- Puzzle 1: Tension between free choice and monotonicity
- Puzzle 2: Know-might and believe-might

Main ingredients of the analysis

- Free choice as ***neglect zero*** effects
- Non truth-conditional analysis of ***might***
- Plausibility model for belief in epistemic logic
- Factivity as presupposition

An epistemic extension BSEL

Free choice inferences

- ◊−free choice principle

$$\Diamond(\quad \square \quad) \,\square\, \Diamond \quad \square\, \Diamond$$

- Modal disjunction or ignorance inference:

$$\square \quad \square\, \Diamond \quad \square\, \Diamond$$

(4) a. Baoyu is in Rongguofu or in Ningguofu.

b. □ Baoyu might be in Rongguofu and he might be in Ningguofu.

- Free choice/ignorance inferences under knowledge/belief:

(5) a. Daiyu believes/knows that Baoyu might be/is in Rongguofu or in Ningguofu.

b. □ Daiyu believes/knows that Baoyu might be in Rongguofu and that he might be in Ningguofu.

Puzzle 1: free choice vs monotonicity

Suppose that Daiyu knows China is in Asia. From this innocent premise we will derive the epistemically contradictory conclusion that China is in Asia and China might be in Europe. The steps of the argument can be summarized as follows:

(6) a. Daiyu knows that China is in Asia. (Assumption)
b. Daiyu knows that the China is in Asia or Europe (not in Asia). (Monotonicity)
c. Daiyu knows that China might be in Europe. (Ignorance e ect)
d. China might be in Europe. (Factivity of Knowledge, c)
e. China is in Asia. (Factivity of Knowledge, a)

- The puzzle:

China is in Asia and China might be not in Asia.

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(7)
a. China is in Asia. (Assumption)
b. Daiyu knows that China might be in Asia. (Assumption)
c. Daiyu knows that China might be in Europe or Asia. (Monotonicity)
d. Daiyu knows that China might be in Europe. (Free choice e ect)
e. China might be in Europe. (Factivity of Knowledge, d)

- The puzzle:

  # China is in Asia and China might be not in Asia.

Epistemic contradictions

Why contradictory?

Infelicity of the following sentence (Wittgenstein1953, Veltman1996, Yalcin2007, Mandelkern2019):

(8) # It is not raining but it might be raining

Epistemic contradictions

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A tension between two desiderata:

- Epistemic contradiction: , Å à –
- Non-factivity of *might*: Å à

(Yalcin2007)

Puzzle 2: know-might vs believe-might

└ Believing epistemic contradictions

(9) # John believes that [it is not raining and it might be training].

(10) # John believes that it is not raining but he believes that it might be raining.

It is always incoherent to believe an epistemic contradiction

Puzzle 2: know-might vs believe-might

- Believing epistemic contradictions

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- If we replace *believe* with *know* in the latter clause of (10) then the result is coherent:

(11) John believes that it is not raining but he knows that it might be raining.

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- If we replace believe with know in the latter clause of (10) then the result is coherent:

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However, if `know' entails `believe', this is puzzling (see Hawthorne2016, Mandelkern2019).

Strategy for puzzle 1: free choice in BSML

- In Bilateral state-based modal logic (BSML, Aloni2022) free choice e ects can be derived only for pragmatically enriched formulas: Åˆ - • à Å , Å

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Let p be the proposition \China is in Asia".

Epistemic might

- A model for BSML is a triple M `W; R; V e. Formulas of BSML are interpreted in models M with respect to an information state s b W. Both support (à) and anti-support (â) conditions are speci ed.
- M; s à Å i ¦ w > s §t b R w t x g and M; t à
  M; s â Å i ¦ w > s M; R w â

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- Compared to Å under state-based constraint: have same predictions but di er at epistemic contradictions and locality.

Strategy for puzzle 2: belief is weak

- Know-might believe-might: the entailment from knowledge to belief only applies to factual propositions and should be blocked in the case ofknow-might and believe-might.

Plausibility model in epistemic logic

In logic, a view of belief follows the intuition that we believe those things that hold in the `best' worlds epistemically accessible to us, see (vanBenthem2006, BaltagSmets2006) etc.

- A plausibility ordering ranks the epistemic states, and hence gives much ner gradations.
- Knowledge interpreted w.r.t epistemic states, and belief interpreted w.r.t doxastic states.
- Doxastic states are those most plausible worlds.

Plausibility model

- A plausibility model is a tuple M `W; ; B_w; V e
  - is an equivalence relation, and gives rise to equivalence classes w for each world w > W.
  - B_w is plausibility relations for all worlds w > W. Intuitively, v B_w vœ says that at world w, the agent thinks that v is at least as plausible as vœ.
- The semantics for the doxastic formula is de ned as:

M; w à B i M; v à for all v > M in $_{B_w}$ ˆ w •

where M in $_{B_w}$ ˆ w • ˜ v > w S¦ vœ > w vœ B_w v v B_w vœ •

- Notice that: on this account knowledge entails belief.

Epistemic extension BSEL

Language

' p S ' S' , ' S' - ' SNES B ' S I ' SÆ S' '

- NEis the non-emptiness atom from team logic (Yang&V•aa•nanen2017)
- B ' means that the agent believes' .
- I ' means that according to the information the agent has,' is true.
- Ærepresents epistemicmight.
- ' ' indicates that ' presupposes' .
- Knowledge as a derived notion:

K ˆ I •

Semantics

- An epistemic model for L_E is a tuple $M = \langle W, \sim, B_w, V \rangle$.
  - $\sim$ is an equivalence relation on W, and gives rise to the equivalence class $[w] := \{v \mid v \sim w\}$.
  - B_w is the plausibility ordering among possible worlds. $Min_{B_w}([w]) = \{v \in [w] \mid \neg \exists v' \in [w] (v' B_w v \wedge \neg v B_w v')\}$
- An information state s is a subset of W: $s \subseteq W$. Formulas in our language are interpreted with respect to an information state: $M, s \models \phi$.
- The interpretation $M, s \models \phi$ stands for "formula ϕ is assertable in s" and $M, s \dashv \phi$ stands for "ϕ is rejectable in s"

Semantics

M ;s à p   i   ¦ w >s V ˆw; p•   1
M ;s â p   i   ¦ w >s V ˆw; p•   0

M ;s à   i   M ;s â
M ;s â   i   M ;s à

M ;s à   -   i   §t; t œt 8 t œ= s and M ;t à   and M ;t œà
M ;s â   -   i   M ;s â   and M ;s â

M ;s à   ,   i   M ;s à   and M ;s à
M ;s â   ,   i   §t; t œt 8 t œ= s and M ;t â   and M ;t œâ

M ;s àNE   i   s x g
M ;s âNE   i   s   g

Semantics

Semantics

$M; s$ □ □ iff □s' □ s and $s' \neq$ □ s.t. $M; s'$ □

$M; s$ □ □ iff $M; s$ □

Semantics

M*;s* □ □ iff □*s*′ □ *s* and *s*′ ≠ □ s.t. M*;s*′ □
M*;s* □ □ iff M*;s* □

M*;s* □ [*l*] iff □*w* □ *s* □ [*w*] □
M*;s* □ [*l*] iff □*w* □ *s*: there is a non-empty subset *t* □ [*w*] and *t* □

Semantics

$\mathrm{M};s \square \square$	iff	$\square s' \square s$ and $s' \neq \square$ s.t. $\mathrm{M};s' \square$
$\mathrm{M};s \square \square$	iff	$\mathrm{M};s \square$
$\mathrm{M};s \square [I]$	iff	$\square w \square s \square [w] \square$
$\mathrm{M};s \square [I]$	iff	$\square w \square s$: there is a non-empty subset $t \square [w]$ and $t \square$
$\mathrm{M};s \square [B]$	iff	$\square w \square s$, $Min_{\leq_w}([w]) \square$
$\mathrm{M};s \square [B]$	iff	there is a non-empty subset $t \square Min_{\leq_w}$ and $t \square$

Reflexivity runs into problems for factivity

- If □ is reflexive then
- We have $[I]p$ □ p. But, we have problems with $[I]$□p □ □p:

Reflexivity runs into problems for factivity

- If □ is reflexive then
- We have $[I]p$ □ p. But, we have problems with $[I]$□p □ □p:

s □ $[I]$□p, but s □ □p

- □p within the scope of $[I]$ is interpreted with respect to $[w]$. Whereas □p is interpreted with respect to s.

Factivity as presupposition

- To derive the factivity of knowledge with *might*-sentences, we define knowledge in term of presupposition and information:

$$[K] \;\; := ([I] \;\;)$$

$$\begin{array}{llll} \mathrm{M}; s & \text{iff} & s & \text{and } s \\ \mathrm{M}; s & \text{iff} & s & \text{and } s \end{array}$$

- Intuitively, '*a* knows ' not only means *a* has information of , it also presupposes that is true. Then we have

$$[K] p \quad p$$

- But also we have the projection of factive inferences under negation:

$$\neg[K] p \quad p$$

(12) a. Daiyu doesn't know Baoyu might be in Ningguofu
b. Baoyu might be in Ningguofu

From knowledge to belief

- In our framework, the entailment from knowledge to belief holds only for the □-free fragment of the language:

$$[K] \quad □ \, [B] \quad , \text{ if } \quad \text{ is a □-free formula}$$

- But $[K]□p$ □ $[B]□p$.

$$s \; □ \; [K]□p, \text{ but } s \; □ \; [B]□p$$

- This gives us a solution to Puzzle 2.

Summary of results I

- Epistemic contradiction

  p □ □¬*p* □ □

- Non-factivity of epistemic possibility

  □*p* □ *p*

- Free choice inference

  [□(□)]⁺ □ □ □ □

- Free choice under attitude verbs (Puzzle 1)

  [*K*]/[*B*]/[*I*][□(□)]⁺ □ [*K*]/[*B*]/[*I*]□ □ [*K*]/[*B*]/[*I*]□

- Believing epistemic contradictions (Puzzle 2)

  [*B*]*p* □ [*B*]□¬*p* □ □

  [*B*]*p* □ [*K*]□¬*p* □ □

- Duality

  [*I*]□ ⇔ □*I*□ , □*I*□ := ¬[*I*]¬

  [*B*]□ ⇔ □*B*□ , □*B*□ := ¬[*B*]¬

Summary of results II

- Factivity of knowledge with *might*

  [K]□ □ □

- Factive inference projects under negation

  ¬[K]□ □ □

- Entailment from knowledge to belief

  [K] □ [B] , if is □-free and NE-free

  But [K]□ □ [B]□ , [K][ ]⁺ □ [B][ ]⁺

Conclusion

- Three challenges
  - Tension between free choice and monotonicity
  - Know-might and believe-might
  - Reflexivity is not enough to capture factivity
- Main ingredients in our analysis:
  - Free choice as pragmatic enrichment in BSML (Aloni2022)
  - Expressive analysis of epistemic possibility (Veltman1996, Yalcin2007)
  - Plausibility model for belief in epistemic logic (VanBenthem2006, BaltagSmets2006)
  - Factivity as presupposition (Lasersohn2009, SpectorEgre2015 etc.)

Further discussion

- Multi-agent setting is needed to fully capture perspective-sensitivity of epistemic *might*.
- In a follow-up work, we combine BSEL and epistemic friendship logic [EFL, Seligman et al 2011] to model multi-agent scenarios.
- Devices from EFL are employed for perspectives shifting.
- Faultless disagreement can be formalized.

Future work

- Dynamics: the full projective behaviour of presuppositions in the dynamic semantics in the future.
- Axiomatization of presupposition: the anti-supported condition